NEW SCHEME

05EC046

Reg. No. 4NMLEC0509

First Semester M.Tech. Degree Examination, January/February 2006

LDE/LBI/LDC/LEC

Linear Algebra

Time: 3 hrs.) (Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Given,

$$A = \begin{pmatrix} 1 & 2 & -2 & -4 & 1 \\ 2 & 4 & -3 & -6 & 1 \\ 3 & 6 & -3 & -6 & 1 \\ 4 & 8 & -4 & -8 & 1 \\ 5 & 10 & -12 & -24 & 8 \end{pmatrix}$$

Find the following :

i) The row reduced echelon from of A.

ii) The row rank and nullity of A.

iii) Find the general solution of the system $Ax = 0$.

iv) Find a basis for the row space of A. (15 Marks)

(b) If W_1 and W_2 are subspaces of a vector space V over a field F show that $W_1 + W_2$ is also a subspace of V. (5 Marks)

2. (a) Find the LU decomposition with $l_{ii} = 1$ for the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 & -1 \\ 4 & 5 & 4 & -1 \\ -4 & 1 & 4 & 4 \\ 6 & -3 & 3 & 1 \end{pmatrix}$$

(10 Marks)

(b) Show that the polynomials of degree at most 3 with real coefficients is a vector space over the field of real numbers. (10 Marks)

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3. (a) Let $T : C^2 \to C^2$ be defined as $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 + 4x_2 \\ x_1 - x_2 \end{pmatrix}$

Let $B_1 = \left\{ v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\};\ B_2 = \left\{ e_1 = \begin{pmatrix} 1 \\ i \end{pmatrix};\ e_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \right\}$

where $i = \sqrt{-1}$. Answer the following :

i) Is T a linear operator on C^2?

ii) Find $[T]_{B_1}$ and $[T]_{B_2}$

iii) What is the relation between $[T]_{B_1}$ and $[T]_{B_2}$? (12 Marks)

(b) Let $T : V \to W$ be a linear transformation and $B = \{v_1, v_2, \ldots, v_k\}$ be a basis fo.

i) If T is one-one show that $Tv_1,\ Tv_2, \ldots Tv_k$ is a linearly independent set in W.

ii) If T is onto show that $Tv_1,\ Tv_2, \ldots, Tv_k$ spans W. (3 Marks)

4. (a) Let

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{pmatrix}$$

Answer the following questions :

i) Find the characteristic and minimal polynomials of A.

ii) Is A diagonalizable?

iii) Find projections E_1 and E_2 such $E_1 + E_2 = I$, $\lambda_1 E_1 + \lambda_2 E_2 = A$, $E_1 E_2 = 0_{3\times 3} = E_2 E_1$, where λ_1 and λ_2 are the eigen values of A. (15 Marks)

(b) Let V be an n-dimensional vector space over C. Let $T : V \to V$ be a linear operator. Prove that if T is both diagonalizable and nilpotent, then $T = Z$, the zero operator. (5 Marks)

5. (a) Let $T : C^3 \to C^3$ be defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4x_1 - 2x_2 + 2x_3 \\ -x_1 + 3x_2 + x_3 \\ x_1 - x_2 + 5x_3 \end{pmatrix}$$

$$If\ W_1 = \left\{ x = \begin{pmatrix} \alpha \\ \alpha \\ 0 \end{pmatrix};\ \alpha \in C \right\},\ W_2 = \left\{ y = \begin{pmatrix} \beta \\ \gamma \\ \beta + \gamma \end{pmatrix};\ \beta, \gamma \in C \right\},$$

show that W_1 and W_2 give a T-invariant direct sum decomposition of C^3. (10 Marks)

(b) Given that

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -0 & 0 & 0 \end{pmatrix}$$

is a nilpotent matrix, determine a matrix P such that $P^{-1}AP$ is in Jordan canonic form. (10 Marks)

6. (a) Given the linear operator $T : C^4 \to C^4$ defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 + 6x_2 + 2x_4 \\ 2x_1 + 6x_2 - 2x_3 + 2x_4 \\ -2x_1 + 2x_2 + 4x_3 \\ 6x_1 - 6x_2 - 6x_3 + 8x_4 \end{pmatrix}$$

If the characteristic polynomial is given by $c(\lambda) = (\lambda - 8)^2(\lambda - 2)^2$, and that the eigen spaces of $\lambda_1 = 8$ and $\lambda_2 = 2$ respectively are

$$W_1 = \left\{ x = \begin{pmatrix} \alpha \\ \alpha \\ 0 \\ 0 \end{pmatrix};\ \alpha \in C \right\},\ W_2 = \left\{ y = \begin{pmatrix} \beta \\ 0 \\ \beta \\ 0 \end{pmatrix};\ \beta \in C \right\}$$

Find :

I) The Jordan from of T.

II) An ordered basis B for C^4 such that $[T]_B$ is in Jordan form.

III) A matrix P such that $P^{-1}[T]_B P$ is in Jordan canonical form. **(15 Marks)**

(b) If A is a 7×7 nilpotent matrix with minimal polynomial λ^4, what are the possibilities for the Jordan canonical form? **(5 Marks)**

7. (a) Given

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}, \; b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix}$$

determine

I) a QR factorizaton of A and hence

II) the least-squares solution of $Ax = b$. **(12 Marks)**

(b) Let A be a real $m \times n$ matrix. Show that the Null spaces of A and $A^T A$ are equal, where A^T denotes the transpose of the matrix A. **(8 Marks)**

8. (a) Find the maximum value of a function $Q(x_1, x_2) = 5x_1^2 + 5x_2^2 - 2x_1x_2$, subject to the constraint $x_1^2 + x_2^2 = 1$. Determine a vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ for which the maximum is attained. **(6 Marks)**

(b) Find a singular value decomposition of

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}.$$

(14 Marks)

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M.Tech. Degree Examination, Dec.08/Jan.09
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions***

1 a. Solve the following system of equations:
$x+2y-3z=1$
$2x+5y-8z=4$
$3x+8y-13z=7$ by Gauss elimination method. (06 Marks)

b. Reduce the following matrix:
$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$ to Row reduced Echelon form. (06 Marks)

c. Find the LU factorization with $l_{ii}=1$ for the matrix $A=\begin{bmatrix} 2 & 1 & 1 & -1 \\ 4 & 5 & 4 & -1 \\ -4 & 1 & 4 & 4 \\ 6 & -3 & 3 & 1 \end{bmatrix}$. (08 Marks)

2 a. Express M as a linear combination of the matrices A, B, C where
$M=\begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$, $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$ (06 Marks)

b. Prove that the set $W=\{(x,y,z)/x-3y+4z=0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$. (07 Marks)

c. Prove that the inverse of two subspaces of a vector space V is a subspace of V. Is it true in the case of union of two subspaces? Justify your answer. (07 Marks)

3 a. If α, β, γ are linearly independent in V(F), prove that the vectors α+β, α-β, α-2β+γ are also linearly independent. (06 Marks)

b. Prove that any two bases of a finite dimensional vector space V have the same number of elements. (06 Marks)

c. Let $T:U\rightarrow V$ be a linear map. Then prove that
i) R(T) is a subspace of V.
ii) N(T) is a subspace of U.
iii) T is 1 – 1 iff the null space (N(T)) is a zero subspace. (08 Marks)

4 a. Prove that $T:U\rightarrow V$ of a vector space U to a vector space V over the same field F is a linear transformation if and only if $\forall\ \alpha, \beta \in U$ and $C_1, C_2 \in F$
$T(C_1\alpha+C_2\beta)=C_1T(\alpha)+C_2T(\beta)$ (06 Marks)

b. Find the eigen space of the linear transformation
$T:R^3\rightarrow R^3$ defined by
$T(x, y, z) = (2x+y, y-z, 2y+4z)$ (07 Marks)

c. Find the linear transformation relative to the bases,
$B_1=\{(1,1),(-1,1)\}$, $B_2=\{(1,1,1),(1,-1,1)(0,0,1)\}$ given the matrix $A_T=\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$. (07 Marks)

5 a. Verify Rank-nullity theorem for the linear transformation $T : R^3 \to R^3$ defined by $T(x,y,z) = (x+y, x-y, 2x+z)$. (06 Marks)

b. Let T be a linear transformation from a vector space U to a vector space V then T is non singular iff T is 1 – 1. (07 Marks)

c. $T : R^2 \to R^2$ is given by $T(x,y) = (4x-2y, 2x+y)$. Verify whether T is non singular. Also find its inverse. (07 Marks)

6 a. Find all invariant subspaces of $A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$ viewed as an operator on R^2. (06 Marks)

b. Determine all possible Jordan canonical forms J for a linear operator $T : V \to V$ whose characteristic polynomial $\Delta T = (t-2)^5$ and whose minimal polynomial $m(t) = (t-2)^2$. (07 Marks)

c. Find a least squares solution of the inconsistent system AX = b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. (07 Marks)

7 a. Define an inner product space. Give one example. If V is an inner product space, then prove that for any vectors α, β in V $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$. (06 Marks)

b. Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace of U of R^4 spanned by $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$, $v_3 = (1, -3, -4, -2)$. (07 Marks)

c. Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (07 Marks)

8 a. Find the maximum and minimum value of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $x^T x = 1$. (06 Marks)

b. Make a change of variable $x = py$ that transforms the quadratic form $x_1^2 - 8x_1x_2 - 5x_2^2$ into a quadratic form with no cross product term. (07 Marks)

c. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. (07 Marks)

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M.Tech. Degree Examination, Dec.09/Jan.10
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$. (06 Marks)

b. Using L-U decomposition method solve the system of equations
$6x_1 - 2x_2 - 4x_3 + 4x_4 = 2$
$3x_1 - 3x_2 - 6x_3 + x_4 = -4$
$-12x_1 + 8x_2 + 21x_3 - 8x_4 = 8$
$-6x_1 - 10x_3 + 7x_4 = 43$ (08 Marks)

c. Solve for the system of linear equations
$x_1 - 2x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$-4x_1 + 5x_2 + 9x_3 = -9$. (06 Marks)

2 a. Find the matrix 'P' which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Verify $P^{-1}AP = D$ where 'D' is a diagonal matrix, hence find A^6. (10 Marks)

b. Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. (10 Marks)

3 a. If V is an inner product space, then prove that for any vector α, β in V and any scalar C.
i) $\|c\alpha\| = |c|\,\|\alpha\|$
ii) $\|\alpha\| > 0$ for $\alpha \neq 0$
iii) $|(\alpha/\beta)| \leq \|\alpha\|\,\|\beta\|$
iv) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$. (06 Marks)

b. Prove that every finite dimensional inner product space has an orthonormal basis. (04 Marks)

c. If V is an inner product space and $\beta_1, \beta_2 \ldots \beta_n$ be any independent vector in V, then prove that it is possible to construct orthogonal vectors $\alpha_1, \alpha_2 \ldots \alpha_n$ in V such that for each $k = 1, 2, \ldots n$ the set $(\alpha_1 \ldots \alpha_k)$ is a basis for the subspace spanned by $\beta_1 \ldots \beta_k$. (10 Marks)

4 a. Construct a spectral decomposition of the matrix A that has orthogonal diagonalization.
$$A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}.$$ (06 Marks)

b. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into quadratic form with no cross product terms. (08 Marks)

c. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $X^T X = 1$. (06 Marks)

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8=50, will be treated as malpractice.

5 a. Let V be a n-dimensional vector space over the field F and W an m-dimensional vector space over F. Let B and B′ be ordered bases for V and W. For each linear transformation $T: V \to W$ show that there is a m x n matrix A such that $[T_\alpha]_{B'} = A[\alpha]_B$. (06 Marks)

b. Find the co-ordinates of (2, 3, 4, -1) relative to the ordered basis $B = \{(1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1), (1, 0, 0, 0)\}$ for V_4. (06 Marks)

c. If U and W are two sub – spaces of a finite dimensional vector space V, then $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$. (08 Marks)

6 a. Define $T: V_3 \to V_2$ by the rule $T(x_1, x_2, x_3) = (x_1, -x_2, x_1 + x_3)$. Show that this is a linear map. (06 Marks)

b. Given a matrix $A = \begin{bmatrix} i & -i & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Determine the linear transformation $T: V_3(R) \to V_2(R)$ relative to the basis B_1 and B_2 given by

i) $B_1 = \{(1\ 1\ 1)\ (1\ 2\ 3)\ (1\ 0\ 0)\}$
$B_2 = \{(1\ 1)\ (1\ -1)\}$.

ii) B_1 and B_2 are standard basis of $V_3(R)$ and $V_2(R)$ respectively. (10 Marks)

c. Let T_1 and T_2 be linear operations on R^2 to R^2 defined as follows:
$T_1(x_1, x_2) = (x_2, x_1)$; $T_2(x_1, x_2) = (x_1, 0)$, show that T_1 and T_2 are not commutative. (04 Marks)

7 a. If T is a linear transformation from V into W where V and W are vector spaces over the field F, and V is finite dimensional, then prove that
rank (T) + nullity (T) = dim V. (08 Marks)

b. Let T be an invertible linear transformation on vector space V(F). Then show that $T^{-1}T = TT^{-1} = I$ (06 Marks)

c. Let 'f' be a linear functional on a vector space V(F), then prove the following:

i) $f(0) = 0$ where '0' on LHS is zero vector of V and '0' on RHS is zero element of F

ii) $f(-\alpha) = -f(\alpha)\ \forall\ \alpha \in V$. (06 Marks)

8 a. Find the least square solution of AX = B for $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$. (04 Marks)

b. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^\perp$ is the null space of E and $V = W \oplus W^\perp$. (06 Marks)

c. Let V be a n-dimensional vector space and let W be m-dimensional vector space over F. Show that the space $\perp(V, W)$ of linear transformation has the dimension mn. (10 Marks)

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08EC046

M.Tech. Degree Examination, May/June 2010
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Solve the system of equations
$y - 4z = 8$, $2x - 3y + 4z = 1$, $5x - 8y + 7z = 1$ **(06 Marks)**

b. Find the LU – factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}.$$ **(07 Marks)**

c. Let v be a vector space over the field F. If S is any subset of v, then show that $S^\circ = [L(s)]^\circ$. **(07 Marks)**

2 a. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$$ **(06 Marks)**

b. Diagonalise the matrix $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. **(07 Marks)**

c. Determine the scalar K such that $(KA)^T(KA) = 1$,

where $A = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Is the value unique? **(07 Marks)**

3 a. Let U and V be vector spaces over the field F. Let $\{v_1, v_2, \ldots\ldots v_n\}$ be a basis of V and let $u_1, u_2, \ldots\ldots u_n$ be any vectors. Then prove that there exists a unique linear mapping $T : V \to U$ such that $T(V_i) = u_i$. **(06 Marks)**

b. Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x,y,z) = (x + 2y - z, y+z, x+y-2z)$. Find a basis and dimension of i) the image of T ii) the kernel of T. **(07 Marks)**

c. Let T be a linear operator on R^3 defined by $T(x,y,z) = (2x, 4x - y, 2x + 3y - z)$
i) Show that T is invertible
ii) Find the formulae for T^{-1} and T^2. **(07 Marks)**

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G = Gilbert

4 a. Given a symmetric matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$. Diagonalise this. (10 Marks)

b. Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$. (10 Marks)

5 a. Let V(F) be a finite dimensional vector space and U_1 and U_2 be the two subspaces of V(F). If V(F) is the direct sum of U_1 and U_2 then prove that Dim V = dim U_1 + dim U_2. (10 Marks)

b. Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.

Find the characteristic polynomial and minimal polynomial of this. (10 Marks)

6 a. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$, subjected to the constraint $X^T X = 1$. (10 Marks)

b. Determine all the possible Jordan's canonical forms of a matrix of order 6 whose minimal polynomial is $m(\lambda) = (\lambda - 2)^2$. (10 Marks)

7 a. Find the singular value decomposition of matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. (10 Marks)

b. What are he generalized eigen vectors? Find the eigen vectors and the Jordan form of $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (10 Marks)

8 a. Determine the invariant subspaces of $A = \begin{bmatrix} 2 & -4 \\ 5 & -2 \end{bmatrix}$ viewed as linear operator on

i) IR^2 ii) C^2. (10 Marks)

b. By using the orthogonal projection determine the least square solution to this system of equations Ax = b where

$A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$. (10 Marks)

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